Study of defects in linear FFAG, preliminary steps

Kicks method
Zgoubi developments

1 Closed orbits errors

We first study equivalent kicks for further comparisons with tracking simulations.

Dipolar type of errors due to magnet misalignement and dipole field defects can be approximated by pairs of entrance/exit kicks such that :

$$\theta_{en}/\theta_{ex} = \Delta(Bl)/(B\rho)$$

$\Delta(Bl)$ representing the effect of the imperfection

The kicks equivalent to the defects are calculated using :

– Matrix formalism
– Geometric considerations concerning misaligned magnet

1.1 Use of matrix formalism

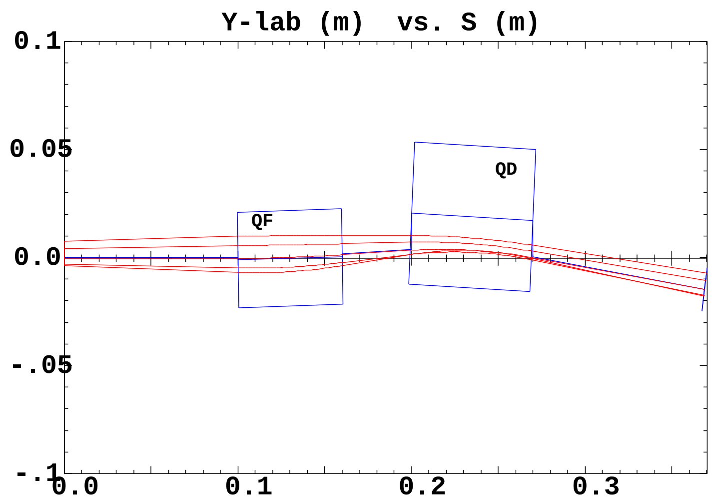


FIG. 1 – EMMA cell.

Combined fonction magnet are represented with the matrix :

– QF, $K > 0 \quad B_{dip} < 0$

$$M_{x,foc} = \begin{bmatrix} \cos\sqrt{K}L & \frac{1}{\sqrt{K}}\sin\sqrt{K}L & \frac{B_{dip}}{g}(\cos\sqrt{K}L - 1) \\ -\sqrt{K}\sin\sqrt{K}L & \cos\sqrt{K}L & -\frac{B_{dip}}{g}\sqrt{K}\sin\sqrt{K}L \\ 0 & 0 & 1 \end{bmatrix} \quad M_{y,foc} = \begin{bmatrix} \cosh\sqrt{K}L & \frac{1}{\sqrt{K}}\sinh\sqrt{K}L & 0 \\ \sqrt{K}\sinh\sqrt{K}L & \cosh\sqrt{K}L & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

– QD, $K < 0 \quad B_{dip} > 0$

$$M_{x,defoc} = \begin{bmatrix} \cosh\sqrt{|K|}L & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}L & \frac{B_{dip}}{g}(\cosh\sqrt{|K|}L - 1) \\ \sqrt{|K|}\sinh\sqrt{|K|}L & \cosh\sqrt{|K|}L & \frac{B_{dip}}{g}\sqrt{|K|}\sinh\sqrt{|K|}L \\ 0 & 0 & 1 \end{bmatrix} \quad M_{y,defoc} = \begin{bmatrix} \cos\sqrt{|K|}L & \frac{1}{\sqrt{|K|}}\sin\sqrt{|K|}L & 0 \\ -\sqrt{|K|}\sin\sqrt{|K|}L & \cos\sqrt{|K|}L & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

x,y,x,y' are coordinates with regard to perfect magnet axis and X,Y,X',Y' with regard to misaligned magnet axis. M_x and M_y are transfert matrix of misaligned magnet :

$$\begin{pmatrix} X_s \\ X_s' \end{pmatrix} = M_x \begin{pmatrix} X_e \\ X_e' \end{pmatrix} \qquad \begin{pmatrix} Y_s \\ Y_s' \end{pmatrix} = M_y \begin{pmatrix} Y_e \\ Y_e' \end{pmatrix}$$

Kicks are placed at entrance and exit of perfect magnet :

$$\begin{aligned} X_e &= x_e & x_s &= X_s & Y_e &= y_e & y_s &= Y_s \\ X_e^{'} &= x_e^{'} + \theta_{xe} & x_s' &= X_s' + \theta_{xs} & Y_e^{'} &= y_e^{'} + \theta_{ye} & y_s' &= Y_s' + \theta_{ys} \end{aligned} \tag{1}$$

M_x (or M_z) are under the form

$$M_x = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

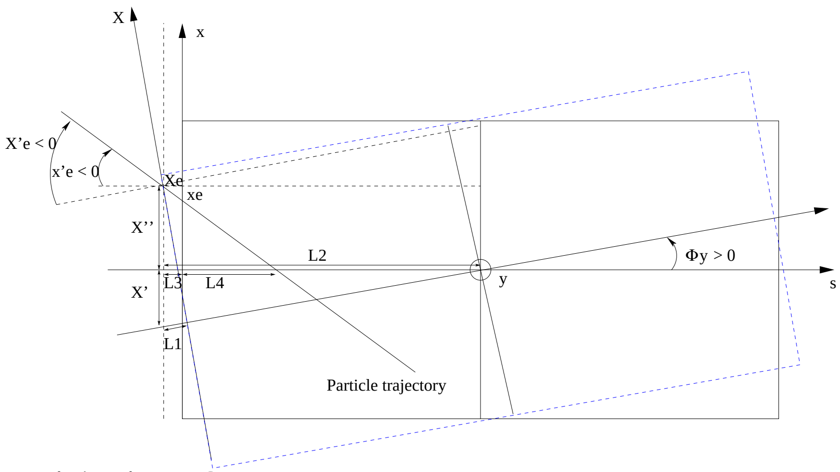
Transfert could be written

$$\begin{bmatrix} x_s \\ x_s' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \theta_{xs} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \theta_{xe} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_e \\ x_e' \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_s \\ x_s' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c + b\,\theta_{xe} \\ d & e & f + e\,\theta_{xe} + \theta_{xs} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_e \\ x_e' \\ 1 \end{bmatrix} \tag{2}$$

we need to find relations (1) for a type of imperfection. As we know matrix $M_{x,z}$ (see previous) we can calculate the new transfert matrix and identify to (2) to extract θ_e, θ_s.

1.2 Example : Vertical rotation ϕ_y



Geometric relations are (with first order approximation) :

$$
\begin{aligned}
X_e &= x_e + \phi_y \tfrac{L}{2} & \qquad x_s &= X_s + \phi_y \tfrac{L}{2} \\
X_e' &= x_e' - \phi_y & \qquad x_s' &= X_s' + \phi_y
\end{aligned}
$$

Considering QF we get :

$$
\begin{bmatrix} x_s \\ x_s' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \phi_y \frac{L}{2} \\ 0 & 1 & \phi_y \\ 0 & 0 & 1 \end{bmatrix} M_{x,foc} \begin{bmatrix} 1 & 0 & \phi_y \frac{L}{2} \\ 0 & 1 & -\phi_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_e \\ x_e' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c - b\phi_y + \frac{L\phi_y}{2} + a\frac{L}{2}\phi_y \\ d & e & f + \phi_y - e + d\frac{L}{2}\phi_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_e \\ x_e' \\ 1 \end{bmatrix}
$$

Identifying 2 et 3 we get :

$$
\begin{aligned}
b\,\theta_{xe} &= -b\,\phi_y + \frac{L}{2}\phi_y(1+a) \\
e\,\theta_{xe} + \theta_{xs} &= \phi_y - e + d\frac{L}{2}\phi_y
\end{aligned}
$$

and finally :

$$
\text{For QF :} \quad \theta_{xe} = -\theta_{xs} = [\frac{\sqrt{K_F}\ \frac{L}{2}}{\tan\sqrt{K_F}\ \frac{L}{2}} - 1]\ \phi_{y_F}
$$

$$
\text{For QD :} \quad \theta_{xe} = -\theta_{xs} = [\frac{\sqrt{|K_D|}\ \frac{L}{2}}{\tanh\sqrt{|K_D|}\ \frac{L}{2}} - 1]\ \phi_{y_D}
$$

We find that entrance and exit kicks have the same amplitude and are equal or opposite

horizontal kicks

Defects :

horizontal displacement	Foc.	$\theta_{xe} = \theta_{xs}$	=	$\sqrt{K}\tan(\sqrt{K}\frac{L}{2})\ \delta x_F$
	Def.	$\theta_{xe} = \theta_{xs}$	=	$-\sqrt{\vert K\vert}\tanh(\sqrt{\vert K\vert}\frac{L}{2})\ \delta x_D$
vertical rotation	Foc.	$\theta_{xe} = -\theta_{xs}$	=	$[\frac{\sqrt{K_F}\ \frac{L}{2}}{\tan\sqrt{K_F}\ \frac{L}{2}} - 1]\ \phi_{y_F}$
	Def.	$\theta_{xe} = -\theta_{xs}$	=	$[\frac{\sqrt{\vert K_D\vert}\ \frac{L}{2}}{\tanh\sqrt{\vert K_D\vert}\ \frac{L}{2}} - 1]\ \phi_{y_D}$
Dipole field defect	Foc.	$\theta_{xe} = \theta_{xs}$	=	$-\frac{B_{dip}}{g}\sqrt{K}\tan(\sqrt{K}\frac{L}{2})\frac{\Delta B}{B_{dip}}$
	Def.	$\theta_{xe} = \theta_{xs}$	=	$\frac{B_{dip}}{g}\sqrt{\vert K\vert}\tanh(\sqrt{\vert K\vert}\frac{L}{2})\frac{\Delta B}{B_{dip}}$

vertical kicks

Defects :

vertical displacement	Foc.	$\theta_{ye} = \theta_{ys}$	=	$-\sqrt{K}\tanh(\sqrt{K}\frac{L}{2})\ \delta y_F$
	Def.	$\theta_{ye} = \theta_{ys}$	=	$\sqrt{\vert K\vert}\tan(\sqrt{\vert K\vert}\frac{L}{2})\ \delta y_D$
horizontal rotation	Foc.	$\theta_{ye} = -\theta_{ys}$	=	$[\frac{\sqrt{K_F}\ \frac{L}{2}}{\tanh\sqrt{K_F}\ \frac{L}{2}} - 1]\ \phi_{y_F}$
	Def.	$\theta_{ye} = -\theta_{ys}$	=	$[\frac{\sqrt{\vert K_D\vert}\ \frac{L}{2}}{\tan\sqrt{\vert K_D\vert}\ \frac{L}{2}} - 1]\ \phi_{y_D}$

1.3 Calcul of kicks for EMMA

E MeV	QD			QF		
	10	15	20	10	15	20
<u>Dév. ang. hor :</u>						
$\theta/\delta x_{D/F}$	-4.60	-3.12	-2.36	7.18	4.67	3.46
$\theta/\phi_{y_{D/F}}$	0.056	0.037	0.028	-0.068	-0.045	-0.034
$\theta/\Delta B/B_{dip}$	-0.077	-0.052	-0.0395	0.049	0.032	0.024
<u>Dév. ang. vert :</u>						
$\theta/\delta y_{D/F}$	5.15	3.36	2.495	-6.275	-4.27	-3.23
$\theta/\phi_{y_{D/F}}$	-0.057	-0.038	-0.028	0.066	0.044	0.033

1.4 Comparison Ray-tracing / Matrix

	10 MeV				
		$x_{e,co}$	$x'_{e,co}$	$y_{s,co}$	$y'_{s,co}$
QF enter		-0.57215	-55.73924	0.0001	0.0000
		$x_{s,co}$	$x'_{s,co}$	$y_{s,co}$	$y'_{s,co}$
Perfect magnet	Ray-tracing	-0.41546	104.584	0.0001397	0.01386
	Matrix	-0.41617	104.380	0.0001406	0.01441
with defect δx_F=10 μm	Ray-tracing	-0.41510	104.696		
	Matrix	-0.41581	104.492		
with defect δy_F=10 μm	Ray-tracing			-0.0002577	-0.12475
	Matrix			-0.0002660	-0.12969

1.5 Numerical simulations of defects

Gaussain distribution of defects, the sigma valuss add quadratically

$$\sigma_{co}^2 = \left[\frac{x}{\delta x}\right]_{QF}\sigma_{\delta x}^2 + \left[\frac{x}{\delta x}\right]_{QD}\sigma_{\delta x}^2 + \left[\frac{x}{\phi_y}\right]_{QF}\sigma_{\phi_y}^2 + \left[\frac{x}{\phi_y}\right]_{QF}\sigma_{\phi_y}^2 + \left[\frac{x}{\Delta B/B}\right]_{QF}\sigma_{\Delta B/B}^2 + \left[\frac{x}{\Delta B/B}\right]_{QD}\sigma_{\Delta B/B}^2$$

Sensitivity coefficient $\left[\frac{x}{\delta x}\right]$ are calculated numerically with a Zgoubi procedure

```
#!/bin/bash
# 1/ genDefect : generate zgoubi.dat with random defects, from defect free
#    structure read in zgoubi_NoDefect.dat.
#    The random seed is read from genDef and renewed (new value stored in
#    genDefect.seed) at end of genDefect
# 2/ run zgoubi using AVEAGEORB and REBELOTE/NREB=100, so to get PU records
#    over 100 turns
# 3/ readPU : reads  PUs from last pass in zgoubi.res, and cumulates (run after
#    run) into readPU.out
# 4/ content of readPU.out is s, <x>, <xp>, <z>, <zp>, PU#,  and can be plotted
#    using zpop/7/20
#
     f77 -o readPU readPU.f
     f77 -o genDefect genDefect.f
#
# 5/ Optionnaly, ./genDefect_IniSeed will force initial seed to given value,
#  this allows checks thanks to identical series of random number series
#
rm readPU.out
rm b_zgoubi*.fai
#
./gebDefect_IniSeed
#
        echo ' '
        echo ' ----------------------------------------------------'
        echo ' COMPUTATION/STORAGE OF C.O. INDUCED BY MAGNET DEFECTS'
        echo ' '
X=1
while [ $X -le 100 ]
do
        echo ' '
        echo ' ----------------'
        echo 'c.o.  computation  #'$X '  follows'
X=$((X+1))
        echo 'c.o.  computation  #'$X
       ~/zgoubi/source/zgoubi
       ./readPU
done
exit
```

1.6 Conclusions

We expect to observe a correlation beetween numerical sensitivity coefficient and amplitude of kicks calculated analytically.

It could yield to compare the relative importance of defects on the closed orbit.

In the future
– Determine tolerance for a chosen σ_{CO}
– Transmission simulations in presence of defects